For triangle lattice 2D Ising model, free energy per-site,

$$F = -kT\left(\ln(2) + \frac{1}{8\pi^2}\int_0^{2\pi}\int_0^{2\pi} \ln\left[\cosh(2K_1)\cosh(2K_2)\cosh(2K_3) + \sinh(2K_1)\sinh(2K_2)\sinh(2K_3) - \sinh(2K_1)\cos(w_1) - \sinh(2K_2)\cos(w_2) - \sinh(2K_3)\cos(w_1 + w_2)\right]dw_1 dw_2\right)$$

Here,

$$K_i = \frac{J_i}{kT} \ (J_i > 0 \ for \ FM)$$

When $J_i < 0 \ and \ |J_1| = |J_2| > |J_3|$, phase transition temperature T_c has the form below,

$$T_c = \frac{2(|J_1| - |J_3|)}{k \cdot \ln\left(1 + \sqrt{1 + e^{-\frac{4|J_3|}{kT_c}}}\right)}$$

① Assuming $\frac{4|J_3|}{kT_c} \gg 1$, then

$$T_c \approx \frac{2(|J_1| - |J_3|)}{k \cdot \ln(2)}$$

② Assuming $\frac{4|J_3|}{kT_c} \ll 1$, then

$$T_c \approx \frac{2(|J_1| - |J_3|)}{k \cdot \ln(1 + \sqrt{2})}$$

In general case,

$$\frac{2(|J_1| - |J_3|)}{k \cdot \ln(1 + \sqrt{2})} < T_c < \frac{2(|J_1| - |J_3|)}{k \cdot \ln(2)}$$