Nonlinear Hall Conductance

- From Boltzmann equation to nonlinear hall

- From Current correlation to nonlinear hall

From Boltzmann equation to nonlinear hall

$$\frac{f - f_0}{-\tau} = \partial_t f + \dot{\boldsymbol{k}} \cdot \boldsymbol{\nabla}_{\boldsymbol{k}} f$$

$$\dot{\boldsymbol{k}} = q\boldsymbol{E}(t) = qE_k e^{iwt}$$

$\Longrightarrow$

$$f = f_0 - \tau\partial_t f - \tau\dot{\boldsymbol{k}} \cdot \boldsymbol{\nabla}_{\boldsymbol{k}} f$$

$$f = f^{(0)} + f^{(1)} + f^{(2)} + \cdots$$

$\Longrightarrow$

$$f^{(0)} = f_0 = \frac{1}{e^{\beta\varepsilon} + 1}$$

$$f^{(1)} = -\tau\partial_t f^{(1)} - \tau\dot{\boldsymbol{k}} \cdot \boldsymbol{\nabla}_{\boldsymbol{k}} f^{(0)}$$

$$f^{(2)} = -\tau\partial_t f^{(2)} - \tau\dot{\boldsymbol{k}} \cdot \boldsymbol{\nabla}_{\boldsymbol{k}} f^{(1)}$$

From Boltzmann equation to nonlinear hall

$$f^{(0)} = f_0$$
$$f^{(1)} = -\tau\partial_t f^{(1)} - \tau q E_k e^{iwt}\partial_k f^{(0)}$$
$$f^{(2)} = -\tau\partial_t f^{(2)} - \tau q E_k e^{iwt}\partial_k f^{(1)}$$

$\Longrightarrow$

$$f^{(1)} = \frac{-q\tau}{1+iw\tau} E_k e^{iwt}\partial_k f^{(0)}$$
$$f^{(2)} = \frac{(q\tau)^2 E_{k_1} E_{k_2} e^{i2wt}}{(1+i2w\tau)(1+iw\tau)}\partial_{k_1}\partial_{k_2} f^{(0)}$$

From Boltzmann equation to nonlinear hall

$$j_\alpha = q \int f(k)\, v_\alpha$$

$$v_\alpha \approx \partial_\alpha \varepsilon - q(\boldsymbol{E} \times \boldsymbol{\Omega})_\alpha$$

$$j_\alpha = j_\alpha^{(0)} + j_\alpha^{(1)} + j_\alpha^{(2)} + \cdots$$

$\Rightarrow$

$$j_\alpha^{(0)} = q \int f^{(0)} \partial_\alpha \varepsilon = 0$$

$$j_\alpha^{(1)} = -q^2 \int f^{(0)} (\boldsymbol{E} \times \boldsymbol{\Omega})_\alpha + q \int f^{(1)} \partial_\alpha \varepsilon$$

$$j_\alpha^{(2)} = -q^2 \int f^{(1)} (\boldsymbol{E} \times \boldsymbol{\Omega})_\alpha + q \int f^{(2)} \partial_\alpha \varepsilon$$

From Boltzmann equation to nonlinear hall

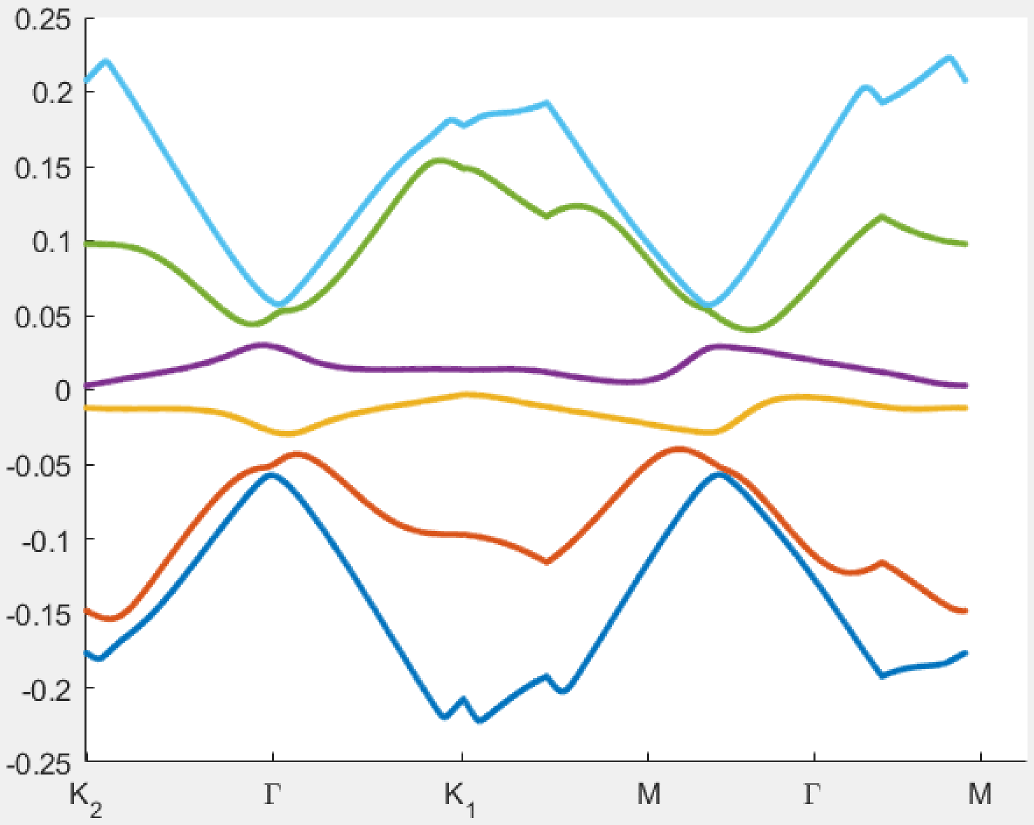
$$j_\alpha^{(1)} = \sigma_{\alpha\beta}\boldsymbol{E}_\beta$$
$$j_\alpha^{(2)} = \chi_{\alpha\beta\gamma}\boldsymbol{E}_\beta\boldsymbol{E}_\gamma$$

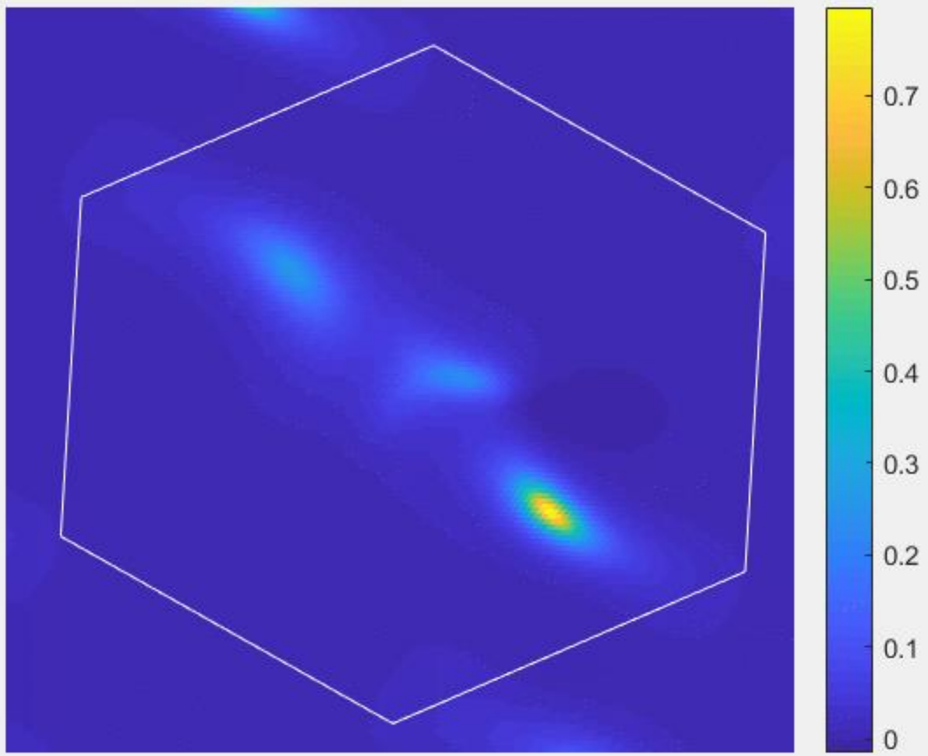
$\Longrightarrow$

$$\sigma_{\alpha\beta} = -q^2\epsilon_{\alpha\beta\gamma}\int f^{(0)}\Omega_\gamma + \frac{q^2\tau}{1+iw\tau}\int f^{(0)}\partial_\alpha\partial_\beta\varepsilon$$

$$\chi_{\alpha\beta\gamma} = -\frac{q^3\tau}{1+iw\tau}\epsilon_{\alpha\beta\eta}\int f^{(0)}\partial_\gamma\Omega_\eta + \frac{q^3\tau^2}{(1+i2w\tau)(1+iw\tau)}\int f^{(0)}\partial_\alpha\partial_\beta\partial_\gamma\varepsilon + (\beta\leftrightarrow\gamma)$$

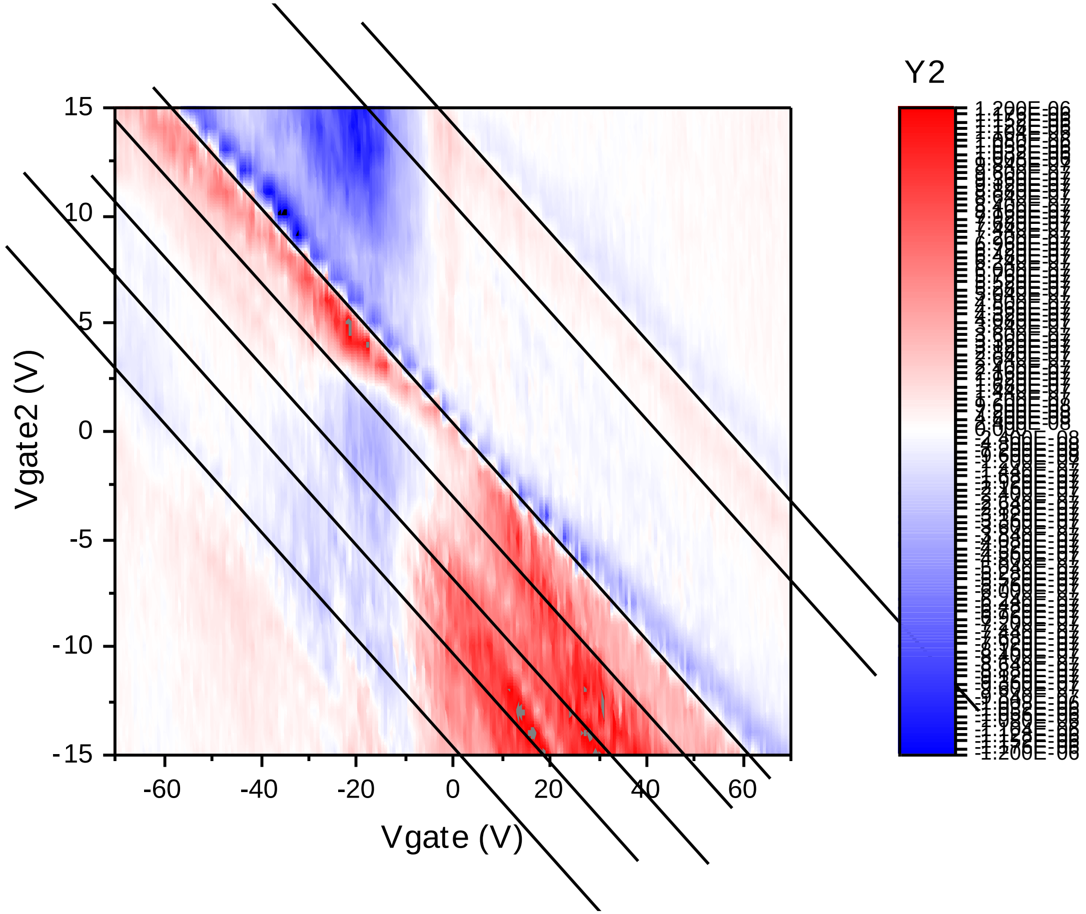
From Boltzmann equation to nonlinear hall





$$\chi_{\alpha\beta\gamma} = -\frac{q^3\tau}{1+iw\tau}\epsilon_{\alpha\beta\eta}\int f^{(0)}\partial_\gamma\Omega_\eta + (\beta \leftrightarrow \gamma)$$

From Boltzmann equation to nonlinear hall



$\epsilon = 0.3\%, \theta = 45°, \alpha = 1.3°, \boldsymbol{p}$ along x direction